Bernouli distribution

Intro

In Bernouli distribution, there are only two cases:

Success, Failure.

We usually use

to present success with probability p.

to present failure with probability q.

Property

If we are convinced to above definition in Intro section, we have these properties:

NOTE

We can consider Bernouli distribution is a special case of binomial distribution with

With degrees of freedom as 2 (k==2)

PDF

if

if

Although they are NOT identical (or exactly said, their formulas are NOT equivalent),

they can be expressed to, since there are only two cases.

for all

for all

Expected Value

Proof:

Variance

Proof:

With these claims,

we can simplify into

Skewness

(P.S. I’m not sure.)

Proof:

We take standardized Bernouli distributed random variable as follows:

With the following claim:

We can simply to:

With higher degrees of freedom

PDF

It is still identical to PDF with k==2. Shown as follows:

Expected Value

And thus,

(For more details, review the above implication)

Variance

Since in Bernouli distribution, we have that

Proof:

Let .

Then

And since

is equal to

which is also equal to the following, by the definition of variance.

The reason why ,

I try to prove it by definition.

which is equal to

(For more details, review the above implication)

Skewness

Proof:

Simply let and use the definition of skewness.

=>

Let’s simplify this term:

First, we expand with cubic formula.

Recall that:

Replace with and with ,we can simply

Then

=

=

=

(since (for more details, see above implicat

ion))

=

Thus,

Ref

For all intro about Bernouil distribution

[Bernoulli distribution - Wikipedia](https://en.wikipedia.org/wiki/Bernoulli_distribution)

For proof of skewness of Bernouli distribution with two degrees of freedom,

[Skewness of Bernoulli Distribution - ProofWiki](https://proofwiki.org/wiki/Skewness_of_Bernoulli_Distribution)